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UNSTEADY PROPAGATION OF A DISK CRACK IN A TRANSVERSELY ISOTROPIC MEDIUM*

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The axisymmetric problem of unsteady disk crack propagation in a transversely isotropic medium is solved. The general problem of dynamic crack motion under the action of given loads is made to close by introducing a fracture criterion. Estimates are presented for the displacements in unloading waves emitted by the growing crack. The results obtained can be utilized to set up a quantitative relation between acoustic emission pulse parameters, the characteristic crack dimension, and the velocity of its motion.

Problems on the motion of cracks with variable velocity in isotropic media were examined in /1-3/.

1. Consider a homogeneous transversely isotropic medium with a crack, loaded along the axis of elastic symmetry by stresses $\sigma(r, t)$. The axisymmetric motion of such a medium obeys the equations

$$C_1^2\left(\Delta u - \frac{u}{r^2}\right) + C_2^2\frac{\partial^2 u}{\partial z^2} + \varkappa C_2^2\frac{\partial^2 w}{\partial r\partial z} - \frac{\partial^2 u}{\partial t^2} = 0 \tag{1.1}$$

$$\varkappa C_2^2\left(\frac{\partial^2 u}{\partial r\partial z} + \frac{1}{r}\frac{\partial u}{\partial z}\right) + C_2^2\Delta w + C_3^2\frac{\partial^2 w}{\partial z^2} - \frac{\partial^2 w}{\partial t^2} = 0$$

$$C_1^2 = \frac{C_{11}}{\rho},\quad C_2^2 = \frac{C_{44}}{\rho},\quad C_3^2 = \frac{C_{33}}{\rho},\quad \varkappa = 1 + \frac{C_{13}}{C_{44}}$$

$$\Delta = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}$$

Here u, w are the components of the displacement vector in the direction of the axes r, z the z axis is directed along the axis of symmetry and is orthogonal to the crack surface, whose initial diameter is $2R_0$, and C_{jk} and ρ are the elastic moduli and mass density. Well-known constraints resulting from the condition for the elastic strain energy to be positive definite are imposed on the constants C_{jk}. The relations required below, which connect the stress and displacement, have the form

$$\sigma_{rz} = \rho C_2^2\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right),\quad \sigma_{zz} = \rho C_2^2(\varkappa - 1)\left(\frac{\partial u}{\partial r} + \frac{u}{r}\right) + \rho C_3^2\frac{\partial w}{\partial z} \tag{1.2}$$

We will represent the general solution of the problem in the form of a sum of the solution of problems on the tension of a medium without a crack and the problem of wave radiation by a crack on whose edges the following stresses are given

$$\sigma_{rz} = 0,\quad \sigma_{zz} = -\sigma_0(r, t) \text{ when } z = \pm 0,\quad 0 \leqslant r < R(t) \tag{1.3}$$

where the velocity of crack propagation is arbitrary, but constrained by the condition $R^{\cdot}(t) < C_R$, where C_R is the Rayleigh wave velocity. The solution of the first problem is trivial, while the solution of the second satisfies the homogeneous initial conditions $u = u^{\cdot} = w = w^{\cdot} = 0$ for $t \leqslant 0$ and the boundary conditions

$$\sigma_{rz} = 0,\ w = 0 \text{ when } z = 0,\ R(t) < r < \infty \tag{1.4}$$

resulting from the symmetry of the problem about the plane $z = 0$. Moreover, satisfaction of the radiation condition and the condition at the edge is necessary to ensure that the solution of the second problem is unique.

We apply the Laplace and Hankel integral transforms to equations (1.1), displaying the difference between the originals and transforms by explicitly writing down the arguments. Consequently, the transform of the components of the displacement vector take the form

$$u(q, z, p) = A_1e^{-\lambda_1|z|} + A_2e^{-\lambda_2|z|} \tag{1.5}$$

$$w(q, z, p) = \operatorname{sign}(z)\,(\mu_1A_1e^{-\lambda_1|z|} + \mu_2A_2e^{-\lambda_2|z|})$$

$$\lambda_{1,2} = [\alpha \pm (\alpha^2 - \beta)^{1/2}]^{1/2},\quad \beta = \nu_1^2\nu_2^2C_2^{-2}C_3^{-2}$$

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$$\alpha = {}^1/_2 C_2^{-2} C_3^{-3} (C_3^2 \nu_1^2 + C_2^2 \nu_2^2 - \varkappa^2 C_2^4), \quad \nu_{1,2}^2 = p^2/q^2 + C_{1,2}^2$$
$$\mu_{1,2} = (\nu_1^2 - C_2^2 \lambda_{1,2}^2)(\varkappa C_3^2 \lambda_{1,2})^{-1} = \varkappa C_2^2 \lambda_{1,2} (C_3^2 \lambda_{1,2}^2 - \nu_2^2)^{-1}$$

We assume the stresses σ_{zz} to be given in the whole $z = 0$ plane; then by using the boundary conditions (1.3) and (1.4) we find the unknown spectral functions A_1 and A_2

$$A_{1,2} = \nu_2 \sigma_{zz} (q, 0, p) [\nu_1^2 + C_2^2 (\varkappa - 1) \lambda_{2,1}^2] [\rho q \lambda_{2,1} \times (\lambda_1 - \lambda_2) R]^{-1} \tag{1.6}$$
$$R (p/q) = C_2 C_3 \nu_1 p^2/q^2 + [C_3^2 \nu_1^2 - (\varkappa - 1)^2 C_2^4] \nu_2$$

From (1.5) and (1.6) we obtain the functional Wiener-Hopf equation

$$\sigma_{zz} (q, 0, p) S (p/q) (q\nu_2)^{-1} + w (q, +0, p) = 0 \tag{1.7}$$
$$S (p/q) = C_2 C_3 \nu_1 \nu_2 (\lambda_1 + \lambda_2) [\rho R (p/q)]^{-1}$$

For our further analysis it is necessary to factorize the function $S (p/q)$ into two factors, one of which has no singularities in the upper half-plane, and the other in the lower half-plane. The functions $\lambda_k (s)$ and $\nu_k (s)$ are analytic in the complex plane $s = p/q$ with two semi-infinite slits $(-i\infty, -iC_{1,2}]$, $[iC_{1,2}, i\infty)$ and two infinite slits $[s_1, \bar{s}_1]$, $[s_2, \bar{s}_2]$, where the branch points $s_{1,2}$ are determined by the inner radical in $\lambda_k (s)$. Let us select those branches of the functions which are positive for $|s| < iC_{1,2}$. The function $(\lambda_1 + \lambda_2)/R (s)$ is analytic in the s plane outside the slits $[iC_2, iC_1]$, $[-iC_1, -iC_2]$. It has simple poles at the points $s = \pm iC_R$ defined by the Rayleigh equation $R (s) = 0$. As $s \to \infty$

$$(\lambda_1 + \lambda_2) R^{-1} (s) \sim \gamma_0 s^{-2} = (C_2^2 + C_3^2) [C_2^2 C_3^2 (C_2 + C_3) s^2]^{-1}$$

Taking this into account and using a Cauchy type integral (transferring the contour of integration to the edges of the slits), $S (s)$ can be represented in the form

$$S (s) = S_+ (s) S_- (s) \gamma, \; \gamma = \gamma_0 C_2 C_3 \rho^{-1} \tag{1.8}$$
$$S_\pm (s) = (s \pm iC_1)^{1/2} (s \pm iC_2)^{1/2} (s \pm iC_R)^{-1} D_\pm (is)$$
$$D_\pm (is) = \exp \left[\frac{1}{\pi} \int_{C_2}^{C_1} g (\tau) (\tau \mp is)^{-1} d\tau \right]$$
$$g (\tau) = \operatorname{arctg} [(a_2 b_1 - a_1 b_2)(a_1 b_1 + a_2 b_2)^{-1}]$$
$$a_{1,2} = \{[c^2 + (C_1^2 - \tau^2)(\tau^2 - C_2^2) C_2^{-2} C_3^{-2}]^{1/2} \pm c\}^{1/2}$$
$$c = {}^1/_2 C_2^{-2} C_3^{-2} [C_3^2 (C_1^2 - \tau^2) - C_2^2 (\tau^2 - C_2^2) - \varkappa^2 C_2^4]$$
$$b_1 = - C_2 C_3 (C_1^2 - \tau^2)^{1/2} \tau^2, \; b_2 = [C_3^2 (C_1^2 - \tau^2) - C_2^4 (\varkappa - 1)^2] (\tau^2 - C_2^2)^{1/2}$$

where the functions S_+, S_- are, respectively, analytic in the upper and lower half-planes of the complex s plane.

2. We will solve (1.7) in combination with conditions (1.3) and (1.4) by Kostrov's method /1/. We introduce the new functions

$$F (q, p) = S_+ (p/q) \sigma_{zz} (q, p), \; G (q, p) = S_-^{-1} (p/q) \gamma^{-1} w (q, p) \tag{2.1}$$
$$\sigma_{zz} (q, p) \equiv \sigma_{zz} (q, 0, p), \; w (q, p) \equiv w (q, +0, p)$$

Then (1.7) can be rewritten as

$$F (q, p) (C_2^2 q^2 + p^2)^{-1/2} + G (q, p) = 0 \tag{2.2}$$

We apply inverse transformations to relations (2.1). Performing the contour integration, we obtain, using the discontinuous Weber-Shafkheitlin integraland the multiplication theorem for the zero-order Hankel function /4/,

$$F (r, t) = \sigma_{zz} (r, t) + \int_0^\pi \int_0^t \left\{ \chi D_+ (C_R) I (\sigma_{zz}, r, C_R) + \int_{C_2}^{C_1} \alpha (s) D_+ (s) I (\sigma_{zz}, r, s) \, ds \right\} d\tau \, d\varphi \tag{2.3}$$

$$\gamma G (r, t) = w (r, t) + \frac{1}{\pi} \int_0^\pi \int_0^t \int_{C_2}^{C_1} \alpha^{-1} (s) D_-^{-1} (-s) I (w, r, s) \, ds \, d\tau \, d\varphi \tag{2.4}$$

Inverting the transforms (2.3) and (2.4), we find

$$\sigma_{zz} (r, t) = F (r, t) + \frac{1}{\pi} \int_0^\pi \int_0^t \int_{C_2}^{C_1} \alpha^{-1} (s) D_+^{-1} (s) I (F, r, s) \, ds \, d\tau \, d\varphi \tag{2.5}$$

$$\gamma^{-1}w(r,t) = G(r,t) + \int_0^{\pi}\int_0^{t}\left\{\chi D_-(C_R)\, I(G,r,C_R) + \int_{C_2}^{C_1}\alpha(s)\, D_-(-s)\, I(G,r,s)\, ds\right\} d\tau\, d\varphi \tag{2.6}$$

From (2.3)-(2.6), in which we have used the following notation

$$I(A,r,s) = \frac{1}{\pi}\int_0^{st} A[\zeta(r), t-\tau]\,\beta(s)\, d\eta \tag{2.7}$$

$$\alpha(s) = (s-C_R)^{-1}(C_1-s)^{1/2}(s-C_2)^{1/2}, \quad \beta(s) = \eta\frac{\partial^2}{\partial\tau^2}(s^2\tau^2-\eta^2)^{-1/2}$$

$$\chi = (C_1-C_R)^{1/2}(C_2-C_R)^{1/2}, \quad \zeta(r) = (\eta^2+r^2-2r\eta\cos\varphi)^{1/2}$$

it follows that

$$F(r,t) = -f(r,t), \; f = \mathbf{L}\sigma_0, \; 0 \leqslant r < R(t) \tag{2.8}$$
$$G(r,t) = 0, \; R(t) < r < \infty$$

The operator $\mathbf{L}$ is defined by (2.3).

Therefore, the problem has been reduced to finding a function $F(r,t)$ from (2.2) and the boundary conditions (2.8).

Let us apply inverse integral transforms to (2.2). We use the Gegenbauer formula /4/

$$J_0(C_2\tau q)\, J_0(rq) = \frac{1}{\pi}\int_0^{\pi} J_0[\varkappa_1(\tau,r)\, q]\, d\varphi, \quad \varkappa_1(\tau,r) = (\tau^2C_2^2+r^2-2\tau C_2 r\cos\varphi)^{1/2}$$

and make the change of variables $\varkappa_1(t, r_0) = r$. We hence have

$$\int_0^{t_0}\int_{|r_0-C_2(t_0-t)|}^{r_0+C_2(t_0-t)} F(r,t)\,[C_2^2(t_0-t)^2-(r_0-r)^2]^{-1/2}\,[(r_0+r)^2-C_2^2(t_0-t)^2]^{-1/2}\, r\, dr\, dt = -{}^1/_2\pi G(r_0,t_0) \tag{2.9}$$

Let us consider the case when $r_0 \geqslant C_2t_0 > R(t_0)$. We introduce the characteristic variables $\xi = C_2t - r$, $\eta = C_2t + r$. Then the equation $\eta = R_*(\xi)$, or $\eta - \xi = 2R\,[{}^1/_2C_2(\eta+\xi)]$, determines the law of crack edge motion in (ξ,η) coordinates.* We use the notation $\eta_1 = \eta - C_2t_0$, $\xi_1 = C_2t_0 - \xi$ and we rewrite (2.9) in the form

$$\int_{r_0}^{C_2t_2-\xi_R}(\xi_1^2-r_0^2)^{-1/2}\, d\xi_1 \int_{\xi_1-C_2t_0}^{\eta_0} F(C_2t_0-\xi_1, C_2t_0+\eta_1)(\xi_1+\eta_1)(r_0^2-\eta_1^2)^{-1/2}\, d\eta_1 = 0 \tag{2.10}$$

for $\eta_0 > R_*(\xi)$. With respect to the inner integral, (2.10) is an Abel integral equation. Solving it for ξ_1, we obtain an equation of the same type. Inverting this for η_1 we express the function F on the continuation of the crack in terms of its value f on the crack

$$F(\xi_0,\eta_0) = \frac{1}{\pi r_0}\frac{d}{dr_0}\int_{R_*(\xi_0)-C_2t_0}^{r_0}(r_0^2-\nu^2)^{-1/2}\,\nu\, d\nu \int_{-\xi_0-C_2t_0}^{R_*(\xi_0)-C_2t_0} f(\xi_0, \eta_1+C_2t_0)(\eta_1+C_2t_0-\xi_0)(\nu^2-\eta_1^2)^{-1/2}\, d\eta_1 \tag{2.11}$$

The integral with respect to ν can be evaluated explicitly. Differentiating the remaining integral with respect to the parameter r_0 and changing to physical variables, we find

$$F(r_0,t_0) = \frac{1}{\pi\sqrt{r_0-R(t_1)}}\int_{r_0-C_2t_0}^{R(t_1)} f\left(r, t_0-\frac{r_0-r}{C_2}\right) \times \frac{(R(t_1)-r)^{1/2}}{r_0-r}\left(\frac{r}{R(t_1)}\right)^{1/2} dr \tag{2.12}$$

where t_1 is the solution of the equation

$$C_2t_1 - R(t_1) = C_2t_0 - r_0 \tag{2.13}$$

The relationship (2.12) agrees with the corresponding equation from /1/ to within the radical in the integrand.

3. It was assumed when formulating the problem that the law of crack motion is given arbitrarily. To solve the problem of crack propagation under the effect of given forces, the asymptotic form of the stresses at the crack edges must be determined. We set $R(t_1) \sim R(t_0) - R^{\boldsymbol\cdot}(t_0)\,(t_0-t_1)$ as $r_0 \to R(t_0)+0$. Using (2.13), we find

$$t_1 \sim t_0 - [r_0 - R(t_0)]\,[C_2 - R^{\boldsymbol\cdot}(t_0)]^{-1}$$
$$R(t_1) \sim r_0 - C_2\,[r_0 - R(t_0)]\,[C_2 - R^{\boldsymbol\cdot}(t_0)]^{-1}$$

Substituting these estimates into (2.12), we obtain

$$F(r_0,t_0) \sim K_f(R^{\boldsymbol\cdot})\,\{2\pi\,[r_0 - R(t_0)]\}^{-1/2} \tag{3.1}$$

$$K_f(R^\cdot)=\left(\frac{2}{\pi}\right)^{1/2}\left[1-\frac{R^\cdot(t_0)}{C_2}\right]^{1/2}\int\limits_0^{C_2t_0} f\left[R(t_0)-r,\ t_0-\frac{r}{C_2}\right]\times\left[1-\frac{r}{R(t_0)}\right]^{1/2} r^{-1/2}\,dr$$

Hence, we see that the asymptotic behaviour of $F(r_0, t_0)$ as $r_0 \to R(t_0)+0$ is analogous to the asymptotic form of the stress $\sigma_{zz}(r_0, 0, t_0) \sim K_I(t_0)\{2\pi[r_0-R(t_0)]\}^{-1/2}$ at the crack edge. Using (2.5) and (3.1) we find the dynamic stress intensity coefficient

$$\begin{gathered}K_I(R^\cdot)=K_f(R^\cdot)+\left(\frac{2}{\pi^4}\right)^{1/2}\left[1-\frac{R^\cdot(t_0)}{C_2}\right]^{1/2}\times\\ \int\limits_0^{\pi}\int\limits_0^{t_0}\int\limits_{C_2}^{C_1}\alpha^{-1}(s)\,D_+^{-1}(s)\int\limits_0^{st_0}\int\limits_0^{C_2(t_0-\tau)} f\left[\zeta(R)-r,\ t_0-\tau-\frac{r}{C_2}\right]\times\\ \left[1-\frac{r}{\zeta(R)}\right]^{1/2} r^{-1/2}\,dr\,\beta(s)\,d\eta\,ds\,d\tau\,d\varphi,\quad r_0\to R(t_0)+0\end{gathered}\tag{3.2}$$

where $\alpha(s)$, $\beta(s)$, $\zeta(R)$ and $D_+(s)$ are determined by (2.7) and (1.8), while the variables of integration τ, η are connected by the relationship

$$\tau R^\cdot(t_0) = R(t_0) - [\eta^2 + R^2(t_0) - 2R(t_0)\,\eta\cos\varphi]^{1/2}$$

To close the crack propagation problem, we use the Irwin force criterion

$$K_I(R^\cdot)=K_{ID}(R^\cdot)\tag{3.3}$$

where K_{ID} is the crack resistance under rapid fracture, which is a standard function of the crack velocity for a given material and is determined from experiment /5/ . Here K_{ID} and the energy intensity γ being liberated during crack growth are connected by the relationship

$$\gamma(R^\cdot)=[\Gamma(R^\cdot)/(2C_{44})]\,K_{ID}^2(R^\cdot)$$

The dimensionless function of the crack velocity which becomes unbounded as $R^\cdot(t)\to C_R$ is denoted by $\Gamma(R^\cdot)$. The crack should be stopped when the stress intensity coefficient is $K_I\leqslant \min K_{ID}$. Therefore, (3.3) determines $R=R(t)$ from the action of arbitrary axisymmetric forces applied to its edges.

4. To describe the waves emitted by a moving crack we represent the transform of the displacements (1.5) in the form

$$u(q,z,p) = U(q,z,p)\,\sigma_{zz}(q,0,p),\quad w(q,z,p) = W(q,z,p)\,\sigma_{zz}(q,0,p)\tag{4.1}$$

where the stresses σ_{zz} on the continuation of the crack are determined by (2.3),(2.5) and (2.12). We write the originals of the functions U, W as follows

$$\begin{gathered}U(r,z,t)=\int\limits_0^\infty J_1(qr)\,dq\,\frac{1}{2\pi i}\int\limits_l\sum_{k=1}^2 B_k\exp(-q\lambda_k z+qts)\,ds\\ W(r,z,t)=\int\limits_0^\infty J_0(qr)\,dq\,\frac{1}{2\pi i}\int\limits_l\sum_{k=1}^2 \mu_k B_k\exp(-q\lambda_k z+qts)\,ds\end{gathered}\tag{4.2}$$

where l is the Mellin-Bromwich contour, $\lambda_k(s)$, $\mu_k(s)$, $B_k = qA_k\sigma_{zz}^{-1}$ are determined by (1.5),(1.6), and $s=p/q$. We will estimate the inner integral by the saddle-point method. We consequently obtain

$$\begin{gathered}I(u)\sim q^{-1/2}\sum_{k=1}^2 B_kP_k\cos Q_k,\quad I(w)\sim -q^{-1/2}\sum_{k=1}^2\mu_kB_kP_k\sin Q_k\\ P_k=(1/2\,\pi\,|zf_k''(i\tau_k)|)^{-1/2},\quad Q_k=q\,[t\tau_k-z\lambda_k(\tau_k)]-1/2\,\varphi_0\\ \varphi_0=\arg f_k''(i\tau_k),\quad f_k=st/z-\lambda_k(s)\end{gathered}\tag{4.3}$$

The saddle point $s_k=+i\tau_k$ is determined from the equation $f_k'(s)=0$.

Substituting $I(u)$ and $I(w)$ into (4.2), we obtain integrals with respect to q which can be expressed in terms of Gauss's hypergeometric functions ${}_2F_1(a,b;c;z)$. However, we assume that $qr\gg 1$. Then replacing the Bessel function by an asymptotic formula for large values of the argument, we can write U and W in the form

$$\begin{gathered}U(r,z,t)\sim -\sum_{k=1}^2 P_kB_k\frac{\partial}{\partial r}J_k(r,z,t)\\ W(r,z,t)\sim -\sum_{k=1}^2\tau_k^{-1}\mu_kP_kB_k\frac{\partial}{\partial t}J_k(r,z,t)\end{gathered}\tag{4.4}$$

$$J_k(r, z, t) = \int_0^\infty T_k(q)\, dq, \quad T_k(q) = q^{-1} \cos(Q_k) \cos\left(qr - \frac{\pi}{4}\right)$$

We will assume the frequency passband of the recording apparatus to be constrained to the band $\omega_1 \leqslant \omega \leqslant \omega_2$. Let $\Delta\tau$ be the observation time which is sufficient to record these frequencies, where $\Delta\tau > \max\{2\pi/\omega_1, 2\pi/(\omega_2 - \omega_1)\}$. We shall also assume that $\Delta\tau \ll t$, where t is the time of wave packet propagation to the vibration-receiver. Then we can write

$$J_k(r, z, t) \approx \int_{q_1}^{q_2} T_k(q)\, dq, \quad q_i \sim \frac{\omega_i}{2\pi C_1}$$

instead of the improper integral in (4.4).

Inverting (4.1), we obtain formulas for the displacement in the far zone

$$u(r, z, t) = U(r, z, t) ** \sigma_{zz}(r, 0, t) \tag{4.5}$$
$$w(r, z, t) = W(r, z, t) ** \sigma_{zz}(r, 0, t)$$
$$A(r, t) ** B(r, t) = \frac{1}{\pi} \int_0^t \int_0^\pi \int_0^L A(\eta, \tau) B[(\eta^2 + r^2 - 2r\eta\cos\varphi)^{1/2}, \ t - \tau]\, \eta\, d\eta\, d\varphi\, d\tau$$

The limit of integration $L(t)$ is determined by the boundaries of the domain within whose limits the stresses $\sigma_{zz}(r, 0, t)$ are non-zero.

Note that the relationships obtained are found for times when multiple wave diffraction by the crack edge is not taken into account. For later times (for a more accurate estimate of the energy travelling to the crack edges) the analysis of the problem is more complicated.

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FRACTURE OF BRITTLE BODIES WITH PLANE INTERNAL AND EDGE CRACKS *

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A brittle half-space with one or two symmetric plane edge cracks experiencing plane strain is considered. Compressive stresses act at infinity to cause superposition of opposite edges of the cracks (crack) and their mutual slip. Moreover, biaxial tension at infinity is considered for an unbounded brittle body with a plane crack. The body is under plane strain conditions and is stretched in two mutually orthogonal directions, one of which agrees with the direction of the crack. Limits of applicability are determined for the solutions of these problems by utilizing a well-known fracture criterion /1—4/, and a more general solution is given, resulting in a technical strength condition for the brittle bodies under consideration when there is no mutual displacement of the crack edges.

1. General remarks. The generalization is made on the basis of the following principle /5/: all volumes included with a sphere of diameter

$$\Delta \approx \frac{4}{3} \frac{ET}{S_0^2} \tag{1.1}$$

are equally strong if the maximum (ε_1) and minimum (ε_3) relative elongations of the diameters

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